বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.A./B.Sc./B.Com. Part-III (1+1+1) Examination 2020

3rd Year (Honours)

Subject: MATHEMATICS

Paper: VII

Full Marks: 80 (Theory)
Time: 4 Hours (Theory)

Candiates are required to give their answer in their own words as far as practicable.
Questions are of equal value.

Answer any **one question** from the following:

1. (a) Define half adder and draw a logic circuit for it.

   (b) State the Huntington's postulates of Boolean algebra.

   (c) Write a program either in C or FORTRAN77 to read an n×n matrix and test whether it is orthogonal or not.

   (d) Write a program either in C or FORTRAN77 to swap the values of two variables.

2. (a) Describe Do-Loop in FORTRAN77 with suitable examples. What is the difference between Do-Loop and implied Do-Loop?

   Or

   Explain the ‘while’ loop and 'do-while' loop in C with example.

(b) Draw a flowchart to find a root of $f(x)=0$ by bisection method.

(c) Write an algorithm to test whether an integer is prime or not.

(d) Express OR and AND gates using only NOR gate .

3. (a) State and prove Baye's theorem for conditional probability.

(b) An even number of cards are drawn from a full pack of cards. What is the probability that half of them will be black and half of them will be red ?

(c) Two persons A and B have agreed to meet at a specific spot between 1 p.m. and 2 p.m. It has been mutually agreed that the first one to come will wait for 20 minutes and then leave the place. What is the probability of a meeting between A and B if the arrival of each during the indicated hours can occur at random and also the times of arrival are independent ?

4. (a) Show that the most probable number of successes in a Bernoullian sequence of trials in $f(x)-\lambda e^{-\lambda x},(\lambda>0), x>0$ the integer(s) is determined by the inequality.

$$(n+1)p-1 \le i_m \le (n+1)p$$

(b) Calculate mean and variance of normal distribution using moment generating function.

(c) A random variable X is uniformly distributed over the interval (0,2) . Find the distribution of the quadratic equation $t^2+2t-X=0$.

5. (a) If X and Y are independent variates both uniformly distributed over (0,1), find the distribution of $X+Y$ and X–Y .

(b) Calculate median and mode of the distribution having p.d.f. $f(x)=\lambda e^{-\lambda x},(x>0), x>0$.

(c) A population is defined by the probability density function $f(x,a)=\dfrac{x^{1-1}e^{ax}}{\Gamma(1)a^{1}}(0<x<a)l$ being a known constant. Estimate the parameter by the method of maximum likelihood estimation. Show that the estimate is consistent and unbiased.

6. (a) Define an unbiased and consistent estimate. Prove that the sample mean is always unbiased and consistent estimate of the population mean.

(b) Show that the distribution of the sample is the statistical image of the distribution of the population.

(c) Find the sampling distribution of the sample mean for Poisson distribution.

7. (a) Give an example of a Boolean algebra. Explain the difference between Boolean algebra and algebra of real numbers.

(b) For all $x, y, z \in B$ (a Boolean algebra) prove, not using truth table, that

$$x + xyz + xy^c z^c + yz + y^c z = x + z$$

(c) Define disjunctive normal form of a Boolean function and express the following function in SOP form in smallest number of variables

$$x^c yz + xy^c z^c + x^c yz + x^c y^c z + xy^c z + x^c y^c z^c$$

(d) Explain NOR gate with block diagram and truth tables.

8. (a) Draw a flow chart to integrate a function f(x) in [a, b] by trapezoidal rule.

(b) Write a FORTRAN or C program to evaluate the infinite series

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

(c) Write an algorithm to find the real roots of a quadratic equation with proper massages indicating the character of the roots.

(d) Write a FORTRAN or C program to sort a set of n real numbers using bubble sort.

9. (a) State and prove Tchebycheff's Theorem.

(b) Show that central limit theorem (for equal components) implies Law of Large number (for equal components)

(c) If X, Y are independent χ^2 –variates having m and n degree of freedom respectively, find the distribution of X/Y.

(d) If X is $\gamma(n)$ variate, then show that P(0< X <2n) ? (n-1)/n

10. (a) The first, second and third moments of probability distribution about the point 2 are 1, 16, - 40 respectively. Find the mean, variance and third central moment of probability distribution.

(b) Prove that Schwartz's inequality for expectations, i.e. $\left[E(XY)\right]^2 \leq E\left(X^2\right)E\left(Y^2\right)$ and Hence deduce that $-1 \leq p\left(X,Y\right) \leq 1$.

(c) The joint density function of the random variables X,Y is $f\left(x,y\right)=2\left(0<x<1, 0<y<x\right)$.

Find the marginal and conditional density functions. Compute $P\left(0.25<X<0.75 \middle| Y=0.5\right)$

(d) If X is standard normal variate, then prove that $Y = \frac{1}{2}X^2$ is $\gamma\left(\frac{1}{2}\right)$ variate.

11. (a) Consider a random sample of size n without replacement from a finite population of size N and variance σ^2 . Show that variance of sample mean is ? $\sigma^2\left(N-n\right)/\left(n\left(N-1\right)\right)$.

(b) The variable X is normally distributed with mean 68 cm and s.d. 2.5 cm. What should be the size of the sample whose mean shall not differ from the population mean by more than 1 cm. with probability 0.95. [Given that the area under standard normal curve to the right of the ordinate at 1.96 is 0.25]

(c) Find the condition that the sample mean and the sample variance are uncorrelated.

12. (a) The weights in gram of sample of 12 items are 7, 13, 22, 15, 12, 14, 18, 8, 21, 23, 10, 17 taken at random from its population which is normal having standard deviation 5. Find 95% confidence interval for the mean of the population. [Given that the area under standard normal curve to the right of the ordinate at 1.96 is 0.25]

(b) A die was thrown 120 times and the frequencies of different faces were observed to be the following:

Face	1	2	3	4	5	6
Observed Frequency	25	17	15	23	24	16

Test the hypothesis that the die is fair using a significance level of 0.05.

[Given that $P\left(X^2 > 11.1\right) = 0.05$ for 5 degree of freedom]

(c) A computer while calculating correlation coefficient between two variables X and Y from 25 pairs of observations obtained the following results.

$n = 25$ $\Sigma X = 125$ $\Sigma X^2 = 650$

$\Sigma Y = 100$ $\Sigma Y^2 = 460$ $\Sigma XY = 508$

It was however later observed that at the time of checking it had copied down two pairs as.

X	Y
6	14
8	6

while the correct values are

X	Y
8	15
6	5

Obtained the correct value of the correlation coefficient.